# The Fundamentals of Brushless Permanent Magnet Motor Design – **Part 3**

## Analytical Methods


**By**

**Rakesh Dhawan**

BTech, MSEE, MBA


PEG POWER ELECTRONICS GROUP LLC

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# ANALYTICAL METHODS FOR BRUSHLESS PERMANENT MAGNET MOTORS – PART 3



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## ANALYTICAL METHODS

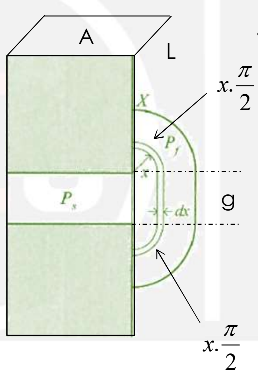
# AGENDA

- Magnetic Circuit Concepts;
  - Air Gap Modeling;
  - Slot Modeling;
- Magnetic Materials
  - Core Loss;
  - Permanent Magnet Magnetic Circuit Model





# AIR GAP MODELING




- The calculation of the air gap permeance using approximation utilizes the fact that permeances add in parallel just as electrical conductances do.

$$P_f = \sum \frac{\mu_o dA}{l} = \sum \frac{\mu_o L dx}{l}$$

Contribution

$$P_f = \int_0^X \frac{\mu_o L}{g + \pi x} dx = \frac{\mu_o L}{\pi} \ln\left(1 + \frac{\pi X}{g}\right)$$

# AIR GAP MODELING

- As we move further from the air gap the contribution of differential permeances decreases.
- The exact value chosen are  not that critical.
- As X increases beyond about 10g, there is little change in the total air gap permeance.

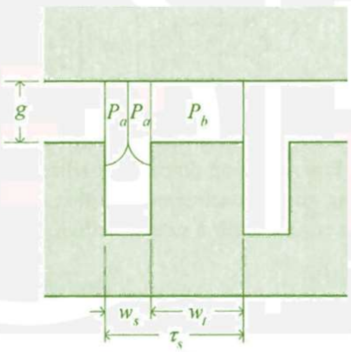




# AIR GAP MODELING

- Discuss its implications with respect to the motor design.
  - Thin Stack Designs
  - Stack Height vs. Air Gap

# SLOT MODELING




$W_s$ = Slot Width
$W_t$ = Tooth Width
$T_s$ = Slot Pitch

- Crude approximation
  - Ignore the flux crossing the gap over the slot

$$P_g = \mu_0 (A - A_s)/g$$

  - $A_s = w_s L$
  - Not accurate





# SLOT MODELING

- Accurate Methods to determine air gap permeance
  - The flux crossing the gap over the slot travels a further distance before reaching the highly permeable material across the gap

$$P_g = \mu_0 . \frac{A}{g_e} \qquad g_e = k_c . g$$

  - $K_c > 1$ is an air gap length correction factor and is known as Carter's coefficient

# SLOT MODELING

- Conformal Mapping Technique
  - Carter was able to determine an analytic magnetic field solution
  - Analytical expressions for Carter's coefficient

$$K_{c1} = \left[1 - \frac{1}{\frac{\tau_s}{w_s}\left(5\frac{g}{w_s}+1\right)}\right]^{-1}$$

$$K_{c2} = \left[1 - \frac{2w_s}{\pi\tau_s}\left\{\tan^{-1}\left(\frac{w_s}{2g}\right) - \frac{g}{w_s}\ln\left[1+\frac{1}{4}\left(\frac{w_s}{g}\right)^2\right]\right\}\right]^{-1}$$

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# SLOT MODELING

- Air gap permeance calculation utilizes the circular arc, straight line modeling
  - The permeance of the air gap over one slot pitch $\tau_s$

$$P_g = 2P_a + P_b = \mu_o L\left[\frac{w_t}{g} + \frac{4}{\pi}\ln\left(1+\frac{\pi w_s}{4g}\right)\right]$$

$$K_{c3} = \left[1 - \frac{w_s}{\tau_s} + \frac{4g}{\pi\tau_s}\ln\left(1+\frac{\pi w_s}{4g}\right)\right]^{-1}$$

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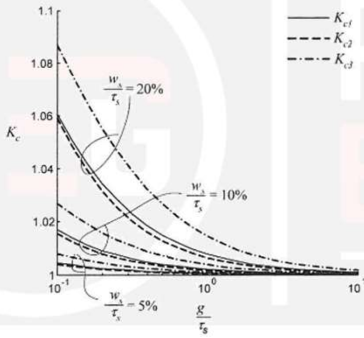
# SLOT MODELING




- $K_{c3}$ dictates a larger correction factor than either of the historical Carter's coefficient expressions





# SLOT MODELING

- The correction factor **increases** as the slot percentage $w_s/\tau_s$ increases (Slot Width Increases).
- Correction factor **decreases** as the relative gap length $g/\tau_s$ increases (Air Gap Increases).
- **Smaller slot openings and larger air gap lengths** require less correction because the influence of the longer flux path length in the slot area is decreased.

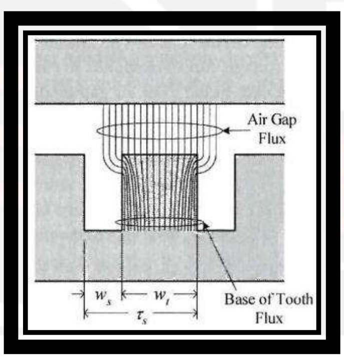
# SLOT MODELING

- The presence of a permanent magnet across the air gap from the slotted structure changes the computation of Carter's coefficient.
- The air gap length g must be replaced by $g+l_m/\mu_R$





# SLOT MODELING




- The presence of slots squeezes the air gap flux into a cross-sectional area $(1-w_s/\tau_s)$ times smaller than the cross-sectional area of the entire air gap over one slot pitch.
- The average flux density $B=\phi/A$ at the base of the teeth is greater by a factor of $\mathbf{(1-w_s/\tau_s)^{-1}}$

# SLOT MODELING EXAMPLE

- Average flux density crossing the air gap is 1.0 T
- Slot fraction, $\alpha_s = w_s/\tau_s$ is 0.5
- The average flux density in the base of the teeth is 2.0 T.
- Since this flux density level is sufficient to saturate most magnetic materials, there is an upper limit to the achievable air gap flux density in a motor.





# WHAT IS AN AIR GAP LENGTH CORRECTION FACTOR?

- Discuss its implications with respect to the motor design
  - Consider Small Air Gap (<0.5mm)
    - Discuss Manufacturing Implications;
    - Discuss Performance Implications.
  - Consider Small Slot Width
    - Repeat the above exercise

# MAGNETIC MATERIAL ANALYSIS

- Assumptions
  - Magnetic field direction;
  - Flux path lengths;
  - Flux uniformity over cross-sectional areas.





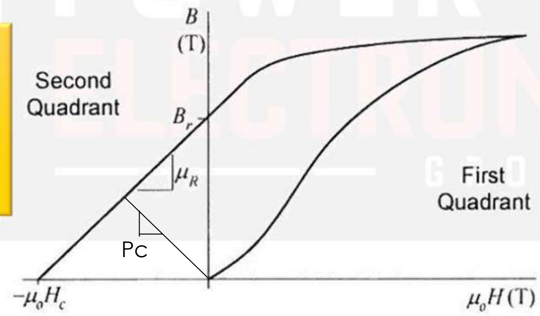
# PERMANENT MAGNETS

- Permanent magnets are magnetic materials with **large hysteresis loops**
  - Alnico
  - Ferrite (ceramic)
  - Somarium-cobalt
  - Neodymium-iron boron (NdFeB)
- Ferrite types are the most popular because they are inexpensive.
- NdFeB magnets are more popular in higher performance applications because they are much cheaper than somarium cobalt

# PERMANENT MAGNETS

- The remanence is the maximum flux density that the magnet can produce by itself

1. Which quadrant is significant for motor design?
2. What is the significance of the first quadrant?








# PERMANENT MAGNETS

- At B=0, the magnitude of the field intensity across the magnet is equal to the negative of the coercivity or coercive force, denoted $H_c$ because $H_c$ is stated as a positive value on permanent magnet specifications.
- The magnitude of the slope of a line drawn from a point on the curve to the origin is known as the permeance coefficient, denoted Pc.
- Pc=0 is operation at the coercivity B=0, H= -Hc, and Pc=infinity is operation at the remanence B=Br, H=0.

# PERMANENT MAGNETS

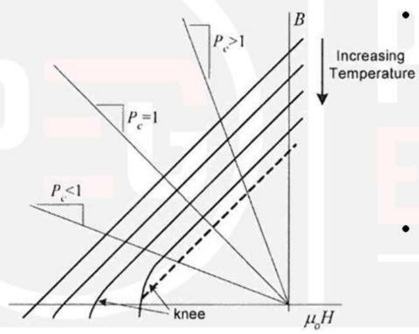
- Permanent magnet materials such as samarium-cobalt and NdFeB materials have straight demagnetization curves throughout the second quadrant at room temperature
- The slope of the straight line demagnetization curve in the second quadrant is equal to $\mu_R$, where $\mu_R$ is the **relative recoil permeability** of the material.

Do Magnets store energy? Why is this energy not dissipated like in a battery?





# PERMANENT MAGNETS




- At higher temperatures, the demagnetization curve shrinks toward the origin, the flux available from the magnet drops, reducing the performance of the magnet.
- This performance degradation is reversible as the demagnetization curve returns to its former shape as temperature drops

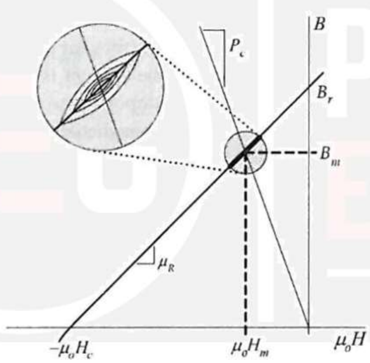
# PERMANENT MAGNETS

- The maximum energy product (BH)max of a magnet is the maximum product of the flux density and field intensity along the magnet demagnetization curve.
- Even though this product has units of energy, it is not actual stored magnet energy, but rather it is a qualitative measure of a magnet's performance capability in a magnetic circuit.
- By convention, (BH)max is usually specified in the English units of millions of Gauss – Oersteds (MG-Oe).
- 1MG-Oe=7.958 kJ/$m^3$





# PM MAGNETIC CIRCUIT MODEL




- When motor windings are energized, the operating point dynamically varies following minor hysteresis loops about the static operating point

# PM CIRCUIT MODEL

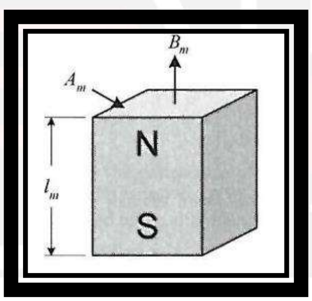
- **Loops:** These loops are thin and have a slope essentially equal to that of the demagnetization characteristic.
- **Trajectory:** The trajectory closely follows the straight-line demagnetization characteristic $B_m = B_r + \mu_R \mu_o H_m$
- **Demagnetization:** If the external magnetic field opposes that developed by the magnet and drives the operating point into the third quadrant past the coercivity, it is possible to irreversibly demagnetize the magnet if a knee in the characteristic is encountered.





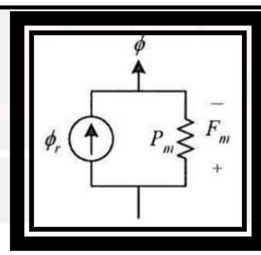
# PM MAGNETIC CIRCUIT MODEL

Rectangular magnet




Flux leaving the magnet is

$$\phi = B_m A_m = B_r A_m + \mu_R \mu_o A_m H_m$$



# PM MAGNETIC CIRCUIT MODEL

$$\phi = \phi_r + P_m F_m$$

$$\phi_r = B_r A_m$$

$$P_m = \frac{\mu_R \mu_o A_m}{l_m}$$

- Uniform Magnetization: It is important to recognize that this model assumes that the physical magnet is uniformly magnetized over its cross section and is magnetized in its preferred direction of magnetization

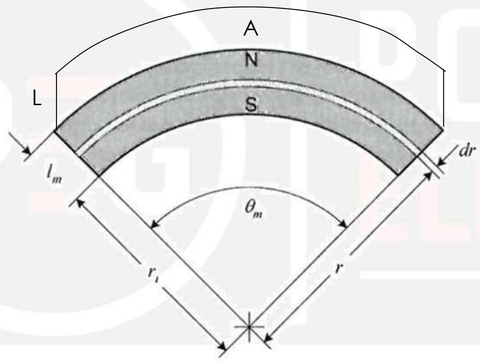




# PM CIRCUIT MODEL

- **Uniformity:** During magnetization the same amount of flux magnetizes each differential length.
- **Br:** The achieved remanence decreases linearly with increasing radius because the same flux over a increasing area gives a smaller flux density.

# PM MAGNETIC CIRCUIT MODEL




$$dR = \frac{dl}{\mu A} = \frac{dr}{\mu r \theta_m L}$$

- Because reluctances add in series just as resistors do, the net reluctance of the magnet is given by the sum, i.e., integral, of each differential reluctance





# PM MAGNETIC CIRCUIT MODEL

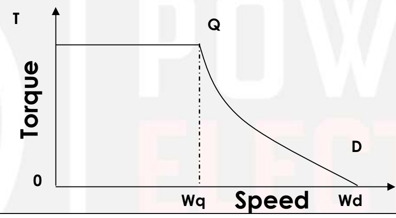
$$R_m = \int_{r_i}^{r_i+l_m} dR = \int_{r_i}^{r_i+l_m} \frac{1}{\mu_R \mu_o L \theta_m r} dr = \frac{\ln\left(1+{l_m}/{r_i}\right)}{\mu_R \mu_o L \theta_m}$$

$$P_m = \frac{\mu_R \mu_o L \theta_m}{\ln\left(1+{l_m}/{r_i}\right)}$$

$$P_m = \frac{\mu_R \mu_o L \theta_m r_i}{l_m}$$

- Which is equivalent to the permeance of a rectangular block having width $\Theta_m r_i$ and length $l_m$. That is, the magnet appears to have a constant width given by the arc width at $r_i$

# TWO PHASE MOTOR EQUATIONS




$W_q$ = Base Speed
$I_d$ = d-axis applied Current
$I_q$ = q-axis applied Current
$I_c$ = Controller Current
$V_q$ – q-axis applied Voltage
$V_d$ = d-axis applied Voltage

$$\text{At Wq}, I_d = 0; I_q = I_c; V_q = E_{qo}; V_d = -X_{so} I_c$$

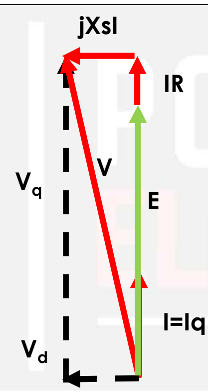
$$\text{At Wd}, \quad I = I_d = -I_c = \frac{V_c - xE_{qo}}{xX_{so}}$$





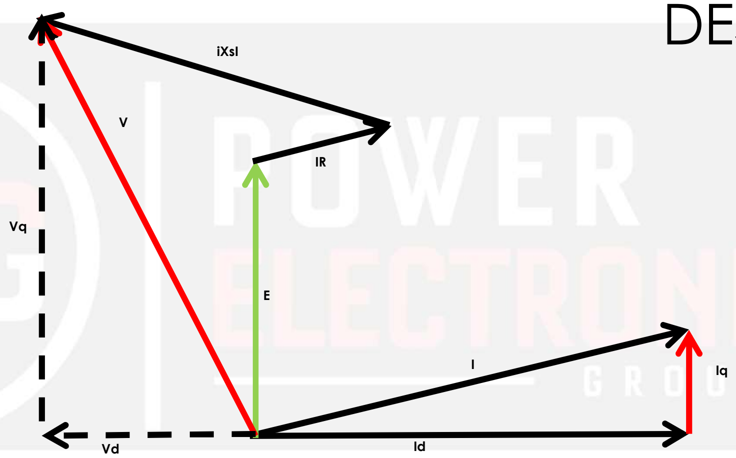
# DESIGN

$$\text{At Wq}, I_d = 0; I_q = I_c; V_q = E_{qo}; V_d = -X_{so} I_c$$




$$V = E + (R + jX_s)I$$

# DESIGN

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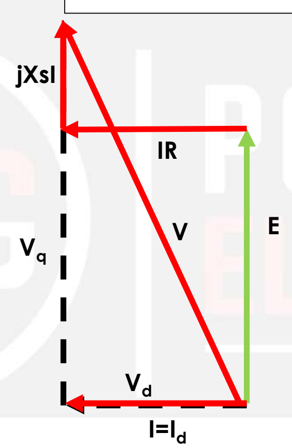
# DESIGN

$$\text{At Wd,} \quad I = I_d = -I_c = \frac{V_c - xE_{qo}}{xX_{so}}$$




$$V = E + (R + jX_s)I$$

## DESIGN

At Wq,

$$I_d = 0$$
$$I_q = I_c$$
$$V_q = E_{q0}$$
$$V_d = -X_{so} I_c$$
$$I = jI_q = jI_c$$
$$V_c^2 = E_{q0}^2 + X_{so}^2 I_c^2$$
$$I_c = \sqrt{\frac{V_c^2 - E_{qo}^2}{X_{so}^2}}$$





## DESIGN

At Wd,

$$I = I_d = -I_c = \frac{V_c - xE_{q0}}{xX_{so}}$$

$$x = \frac{W_d}{W_q}$$

# DESIGN

For a given speed N, rpm

$$w_e = \frac{2.\pi.N}{60}.p$$

$$E_o = \lambda.w_e$$

$$X_q = w_e.L_q$$

$$X_d = w_e.L_d$$

**Field Weakening Equation**

$$I_d = \frac{-\lambda L_d + \sqrt{\lambda^2 L_d^2 + \left(L_q^2 - L_d^2\right) - \left(\lambda^2 - \left(\frac{V_{ph}}{w_e}\right)^2 + I_{cmax}^2 L_q^2\right)}}{L_d^2 - L_q^2}$$

$$\lambda = \frac{Nd\varphi}{dt} = L\frac{di}{dt}$$





# DESIGN

$$I_q = \sqrt{I_{cmax}^2 - I_d^2}$$

$$I_{cmax}^2 = I_d^2 + I_q^2$$

$$T_{rel} = m.p.\left(L_d - L_q\right)I_d.I_q$$

$$T_{syn} = m.p.\lambda.I_q$$

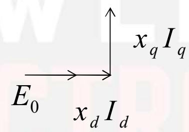
## DESIGN

$$T_{em} = T_{syn} + T_{rel}$$

$$V_{c,rms} = \sqrt{\left(X_q I_q\right)^2 + \left(E_0 + X_d I_d\right)^2}$$

$$I_{c,rms} = \sqrt{I_d{}^2 + I^2{}_q}$$

$$\gamma = \tan^{-1}\frac{I_d}{I_q}$$








## PERFORMANCE

At Base Speed

$I_d, I_q, V_d, V_q$ etc

$$I_d = -I_c \sin(\theta)$$

$$I_q = I_c \cos(\theta)$$

$$V_d = R_{ph} I_d - w_e L_{ph} I_q$$

$$V_q = R_{ph} I_q + w_e L_{ph} I_d + \frac{w_e}{p}\frac{K_w N_{ph} B_g D_r L_{stk}}{\sqrt{2}}$$

$$\delta = \tan^{-1}\left(\frac{V_d}{V_q}\right)$$

# PERFORMANCE

$$V_{ph} = \sqrt{V_d^{\ 2} + V_q^{\ 2}}$$

$$I^2R = m.I_c^{\ 2}R_{ph}$$



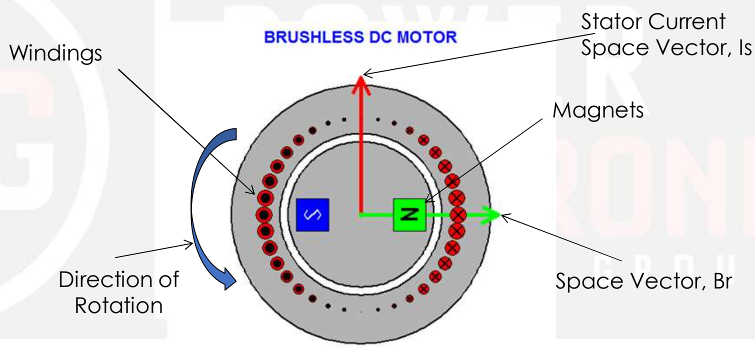






# WHY DOES THE MOTOR TURN?

# WHY DOES THE MOTOR ROTATE?








# ANIMATION

- http://www.ece.umn.edu/users/riaz/animations/brushlessdc.html

# QUESTIONS

- How is the current distributed?
- What are the two vectors?
- Which vector is leading and what is the significance of that?
- How can you physically create this space relationship?

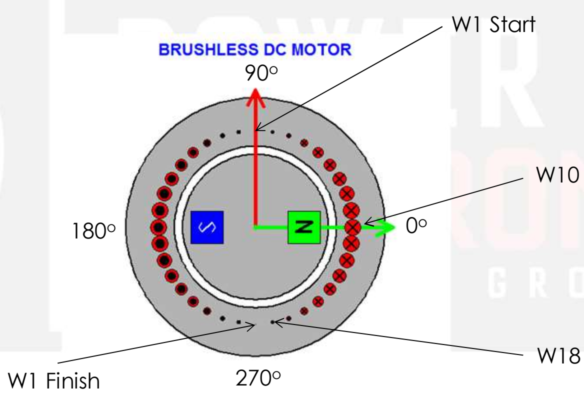




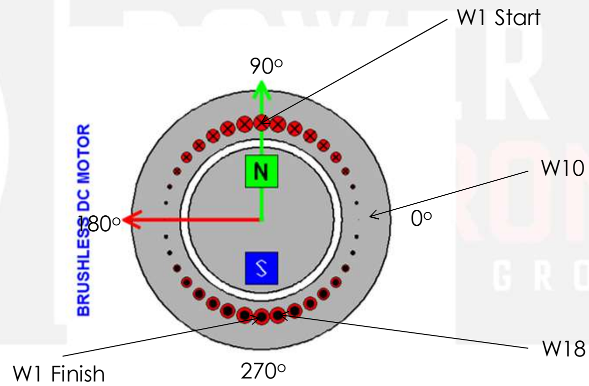
# LET US ANALYZE

**Beware of 0°**

**North Follows the Max Current**



# LET US ANALYZE



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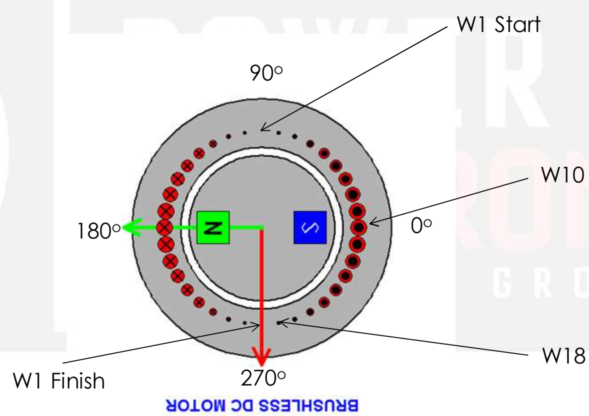
# LET US ANALYZE



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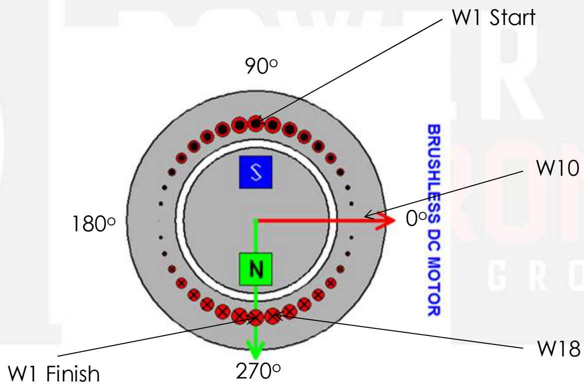
# LET US ANALYZE












# AN INTRODUCTION TO SPACE VECTORS

# SPACE VECTOR ANIMATIONS

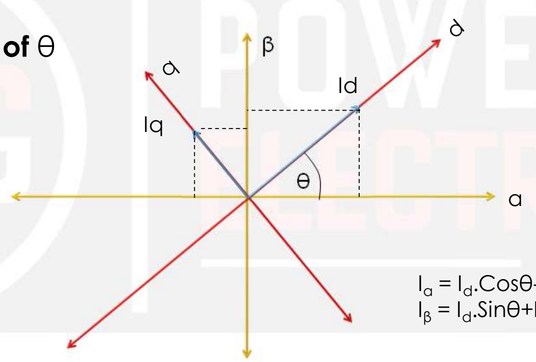
- Motion of Space Vectors
  - http://www.ece.umn.edu/users/riaz/animations/spavecdqclip.html
- Space Vector Representation of the MMF Distribution:
  - http://www.ece.umn.edu/users/riaz/animations/spacevectors.html
- Wave Space Distributions:
  - http://www.ece.umn.edu/users/riaz/animations/sinwaves0.html





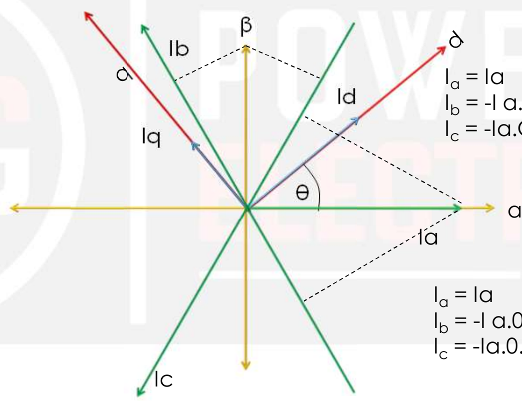
# SPACE VECTOR FUNDAMENTALS

**Beware of θ**




$I_\alpha = I_d.Cos\theta - I_q.Sin\theta$
$I_\beta = I_d.Sin\theta + I_q.Cos\theta$

# SPACE VECTOR FUNDAMENTALS




$I_a = Ia$
$I_b = -I\,a.Cos60+I\beta.Cos30$
$I_c = -Ia.Cos60-I\beta.Cos30$

$I_a = Ia$
$I_b = -I\,a.0.5+I\beta.0.87$
$I_c = -Ia.0.5-I\beta.0.87$





# SPACE VECTOR EQUATIONS

$I_a = I_d.Cos\theta-I_q.Sin\theta$
$I_\beta = I_d.Sin\theta+I_q.Cos\theta$

$I_a = Ia$
$I_b = -I\,a.0.5+I\beta.0.87$
$I_c = -Ia.0.87-I\beta.0.5$

$I_a = I_d.Cos\theta-I_q.Sin\theta$
$I_b = -(I_d.Cos\theta-I_q.Sin\theta).0.5+(I_d.Sin\theta+I_q.Cos\theta).0.87$
$I_c = -(I_d.Cos\theta-I_q.Sin\theta).0.5-(I_d.Sin\theta+I_q.Cos\theta).0.87$

# SPACE VECTOR EQUATIONS

$$\begin{bmatrix} Ia \\ Ib \\ Ic \end{bmatrix} = \begin{bmatrix} Sin\theta & Cos\theta \\ Sin(\theta - \frac{2.\pi}{3}) & Cos(\theta - \frac{2.\pi}{3}) \\ Sin(\theta + \frac{2.\pi}{3}) & Cos(\theta + \frac{2.\pi}{3}) \end{bmatrix} . \begin{bmatrix} Id \\ Iq \end{bmatrix}$$



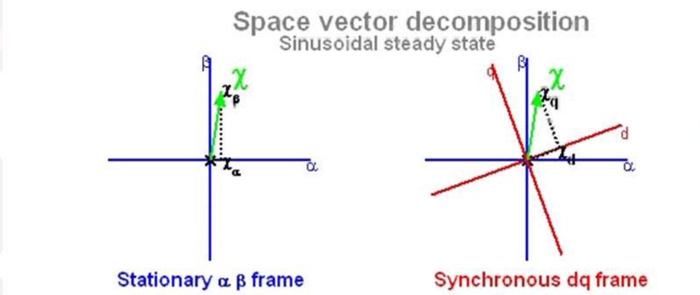


# SPACE VECTOR EQUATIONS

**Beware of θ**

$$\begin{bmatrix} Id \\ Iq \end{bmatrix} = \frac{2}{3} \begin{bmatrix} Sin\theta & Sin(\theta - \frac{2\pi}{3}) & Sin(\theta + \frac{2\pi}{3}) \\ Cos\theta & Cos(\theta - \frac{2\pi}{3}) & Cos(\theta + \frac{2\pi}{3}) \end{bmatrix} . \begin{bmatrix} Ia \\ Ib \\ Ic \end{bmatrix}$$

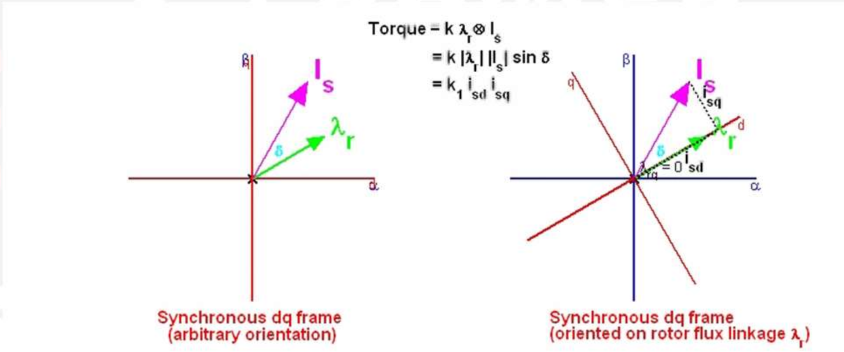
# SINUSOIDAL STEADY STATE





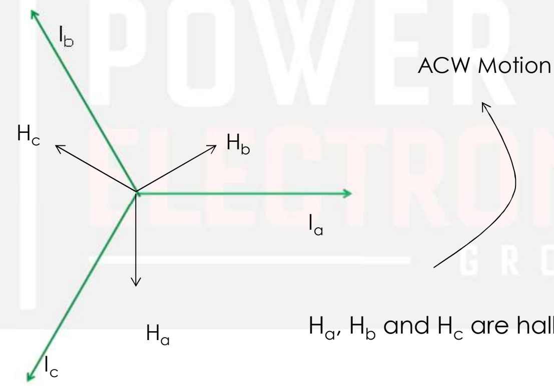



# SPACE VECTORS



## SPACE VECTOR FUNDAMENTALS

Set up the equations such that Iq must lead Id




$H_a$, $H_b$ and $H_c$ are hall sensor signals





## SPACE VECTOR FUNDAMENTALS

Set up the equations such that Iq must lead Id